Dark matter & nonminimal couplings to gravity, from gravitational lensing to gravitational waves

Hong-Yi Zhang

Tsung-Dao Lee Institute, Shanghai Jiao Tong University

https://hongyi18.github.io

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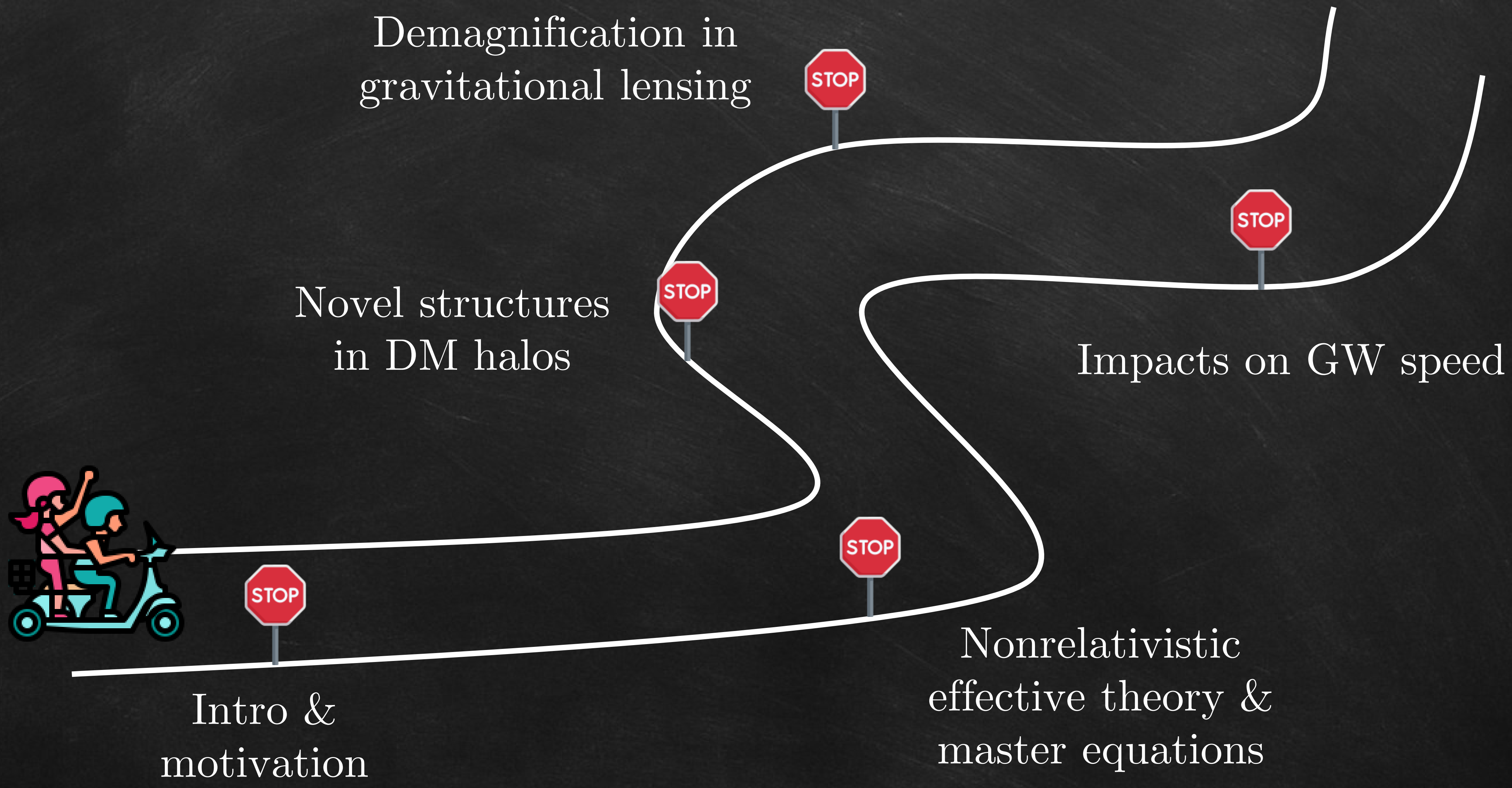
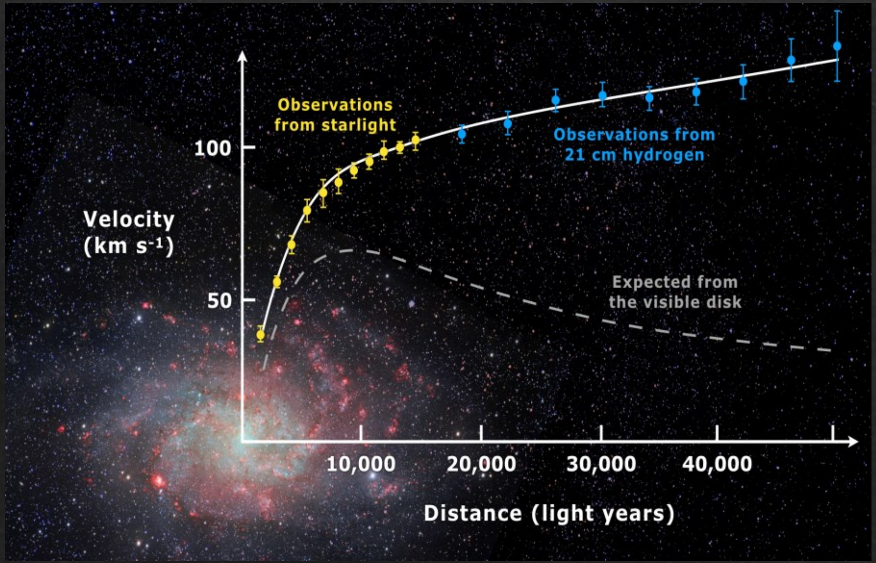
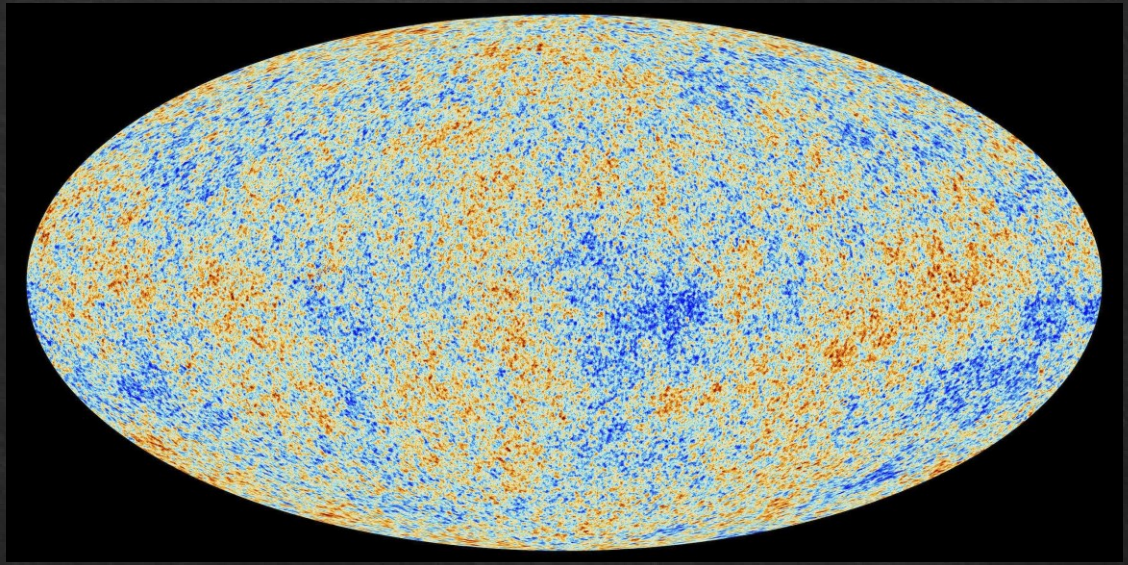
Mainly based on **HYZ** & Ling (JCAP 2023), Chen & **HYZ** (JCAP 2024), **HYZ** (2510.05575)

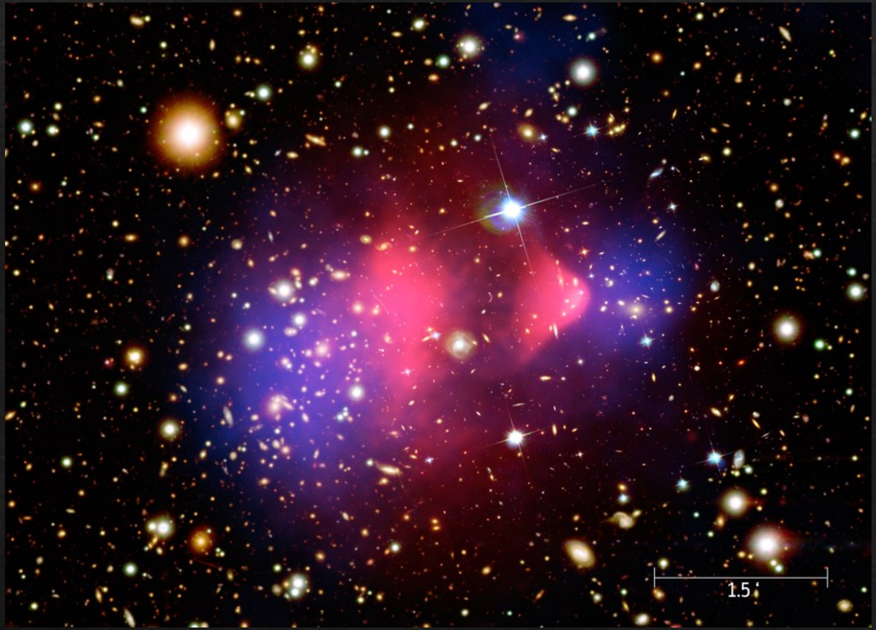
Journey to the future



Evidence for dark matter

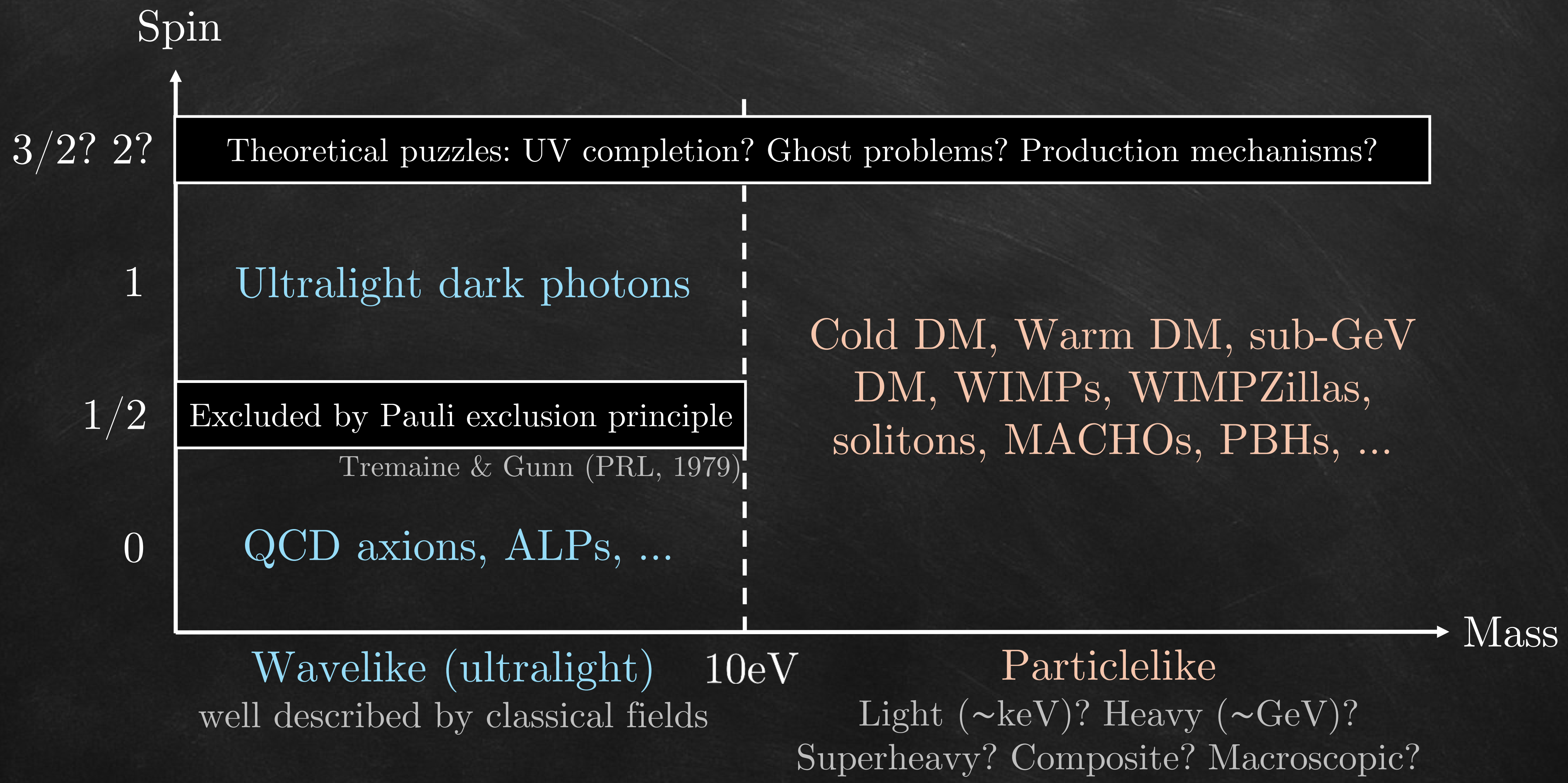




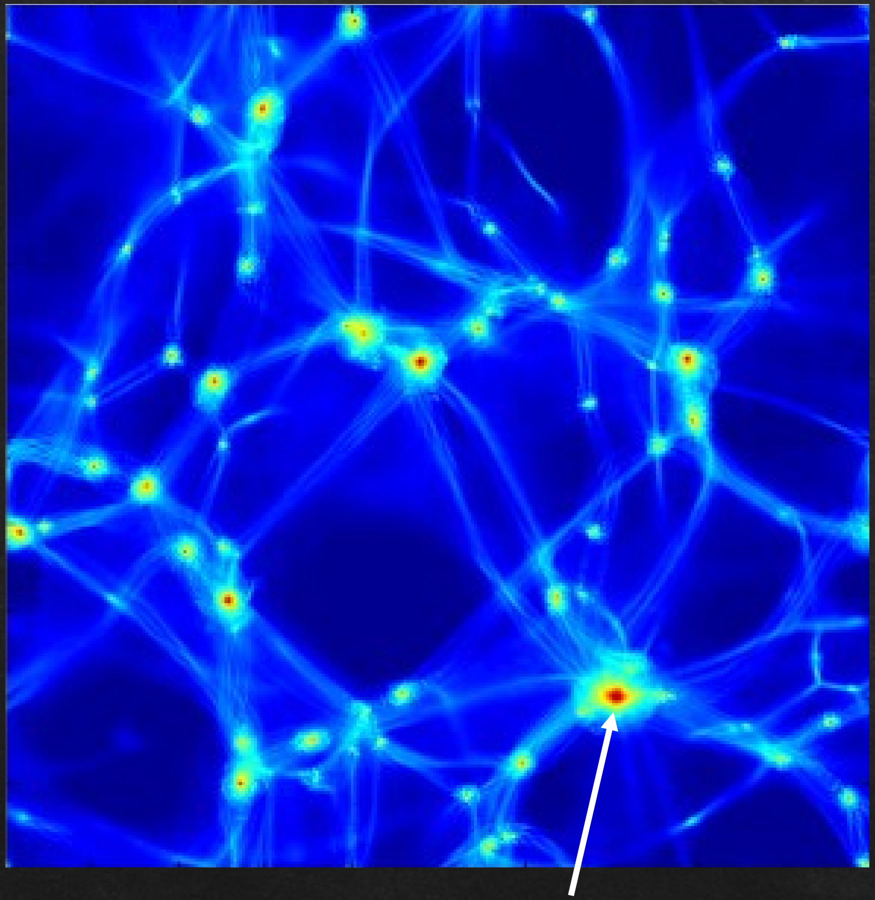


Velocity dispersion of galaxy clusters
Galaxy rotation curves
Gravitational lensing
Bullet cluster
Cosmic microwave background
Large-scale structure
Baryon acoustic oscillations
Type Ia supernovae, ...

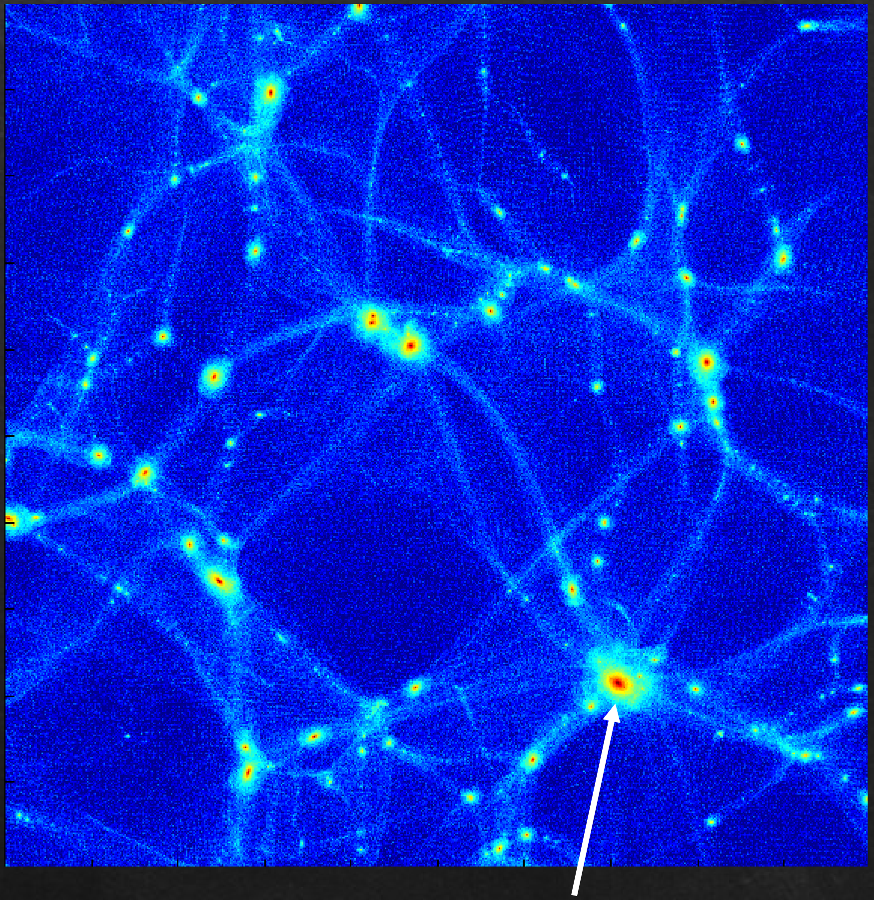
What can be dark matter? A bottom-up view



Wavlike vs. particlelike



Solitons, cored profiles



NFW, cuspy profiles

Schive et al. (Nat. Phys., 2014)

Dark matter interactions

Interactions with other dark fields?
Complicated dynamics—a world of creative and bold ideas!

Gravity
Minimal or not?

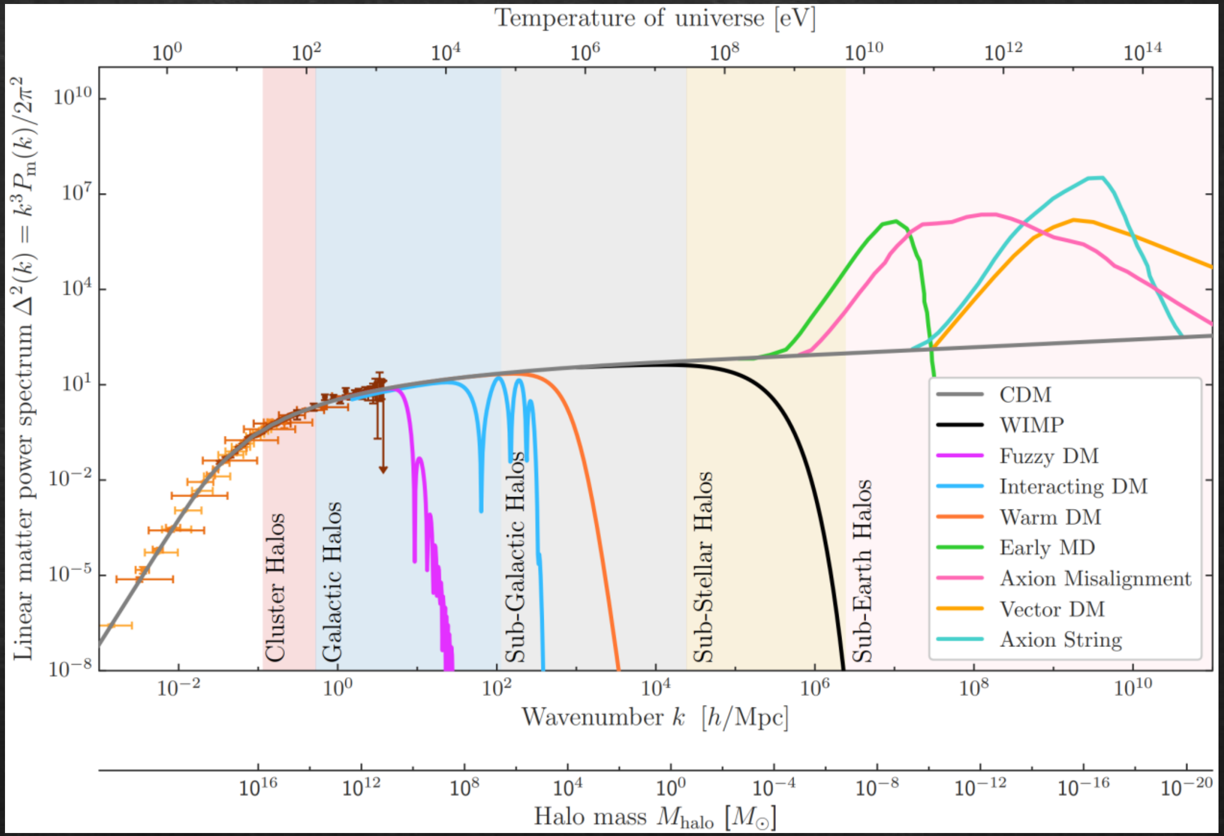
Dark matter

Self-interactions?
Velocity-dependent or not?
How large?

Interactions with baryons?
Accessible to direct detection
Maybe complicated, maybe not, but they usually cost money

Requirements for dark matter

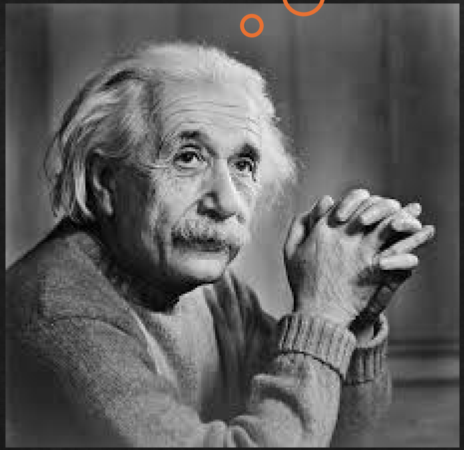
- Cold/warm
  - Thermal candidates $m \gtrsim \mathrm{keV}$
  - Nonrelativistic at MR equality
- Abundance
  - $\Omega_c = 0.265,\ \Omega_b = 0.0493$ (PDG)
- Stability
  - Lifetime $\gtrsim$ 138 billion years
- Reproduce large-scale structure
  - Clusters like pressure-less fluid on large scales $k \lesssim 10\ \mathrm{Mpc}^{-1}$
  - Unconstrained on small scales



Bechtol, et al. (2203.07354)

Nonminimal couplings to gravity

Matter does not directly couple to the Riemann tensor or its contractions.

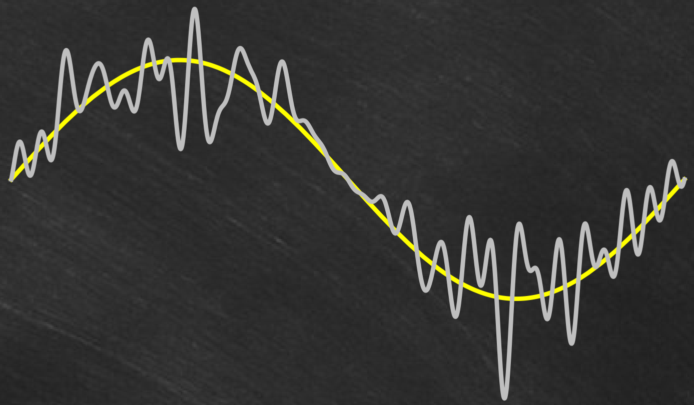


Reasons why we should go beyond it:

- No symmetries forbid them
  (Effective field theory)
- Radiative (quantum) corrections
  (Required for renormalization)
- Rich phenomenology
  (Misalignment, inflation, dark energy, ...)

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset \underbrace{\phi^2 R,}_{\text{Scalar DM}} \; \underbrace{R X_\mu X^\mu, \; R^{\mu\nu} X_\mu X_\nu,}_{\text{Vector DM}} \; \underbrace{\cdots}_{\text{Higher-dimension operators}}$$

Separate slow and fast modes



$$\phi(t, \boldsymbol{x}) = \frac{1}{\sqrt{2m}} \left[\psi(t, \boldsymbol{x}) e^{-imt} + \psi^*(t, \boldsymbol{x}) e^{imt} \right] \longrightarrow \psi(t, \boldsymbol{x}) = \psi_0(t, \boldsymbol{x}) + \underbrace{\text{fast oscillating modes}}_{\text{Perturbations}}$$

$$(\nabla^\mu \nabla_\mu - m^2)\phi + \text{interactions} = 0$$

Klein-Gordon eq + Einstein eqs

Many eqs, very difficult

$$i\partial_t \psi = -\frac{\nabla^2}{2m}\psi + m\Phi\psi + \text{interactions}$$

Schroedinger eq + Poisson eq

A few eqs, so simple!

Salehian, et al. (JHEP, 2020); Salehian, **HYZ,** et al. (JHEP, 2021)

Nonrelativistic effective field theory

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm P}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{m^2}{2} \phi^2 - \frac{\xi}{2} R \phi^2 \right]$$

$$\phi(t, \boldsymbol{x}) = \frac{1}{\sqrt{2ma^3}} \left[\psi(t, \boldsymbol{x}) e^{-imt} + \psi^*(t, \boldsymbol{x}) e^{imt} \right]$$

Integrate out fast-oscillating modes,
keep first-order in perturbations

$$S = \int d^4x \Big[M_{\rm P}^2 a (a^{-2} \Phi \nabla^2 \Phi - 3\dot{a}^2 - 6\ddot{a}\Phi) + i\dot{\psi}\psi^* + \frac{1}{2a^2 m} (\nabla^2 \psi) \psi^* - \frac{m}{a} \Phi |\psi|^2 - \frac{\xi}{m} \left(\frac{\nabla^2 \Phi}{a^3} + 3H^2 + 3\frac{\ddot{a}}{a} \right) \Big]$$

Invariant under U(1)

$$N = \int d^3x \, |\psi|^2$$

Nonrelativistic description: Schroedinger-Poisson equations

Scalar dark matter field

Gravitational potential

Effective self-interactions

$$i\partial_t\psi = -\frac{\nabla^2}{2m}\psi + m\Phi\psi + 4\pi G m \epsilon L^2 \rho_L \psi \qquad m^2\epsilon L^2 = \xi$$

Rest-mass density $\rho = mn$

$\epsilon = \pm 1$, sign of the NMC

NMC strength, a length scale when NMCs become important

$$\nabla^2\Phi = 4\pi G(\rho + \epsilon L^2 \nabla^2 \rho) \equiv 4\pi G \rho_L$$

Corrections to the Newtonian gravity

To be consistent with solar system tests: $L \ll 10\text{kpc}\left(\frac{0.4\text{GeV}/\text{cm}^3}{\rho}\right)^{1/2}\left(\frac{\Phi_\odot}{10^{-6}}\right)^{1/2}$

HYZ (JHEP, 2025)

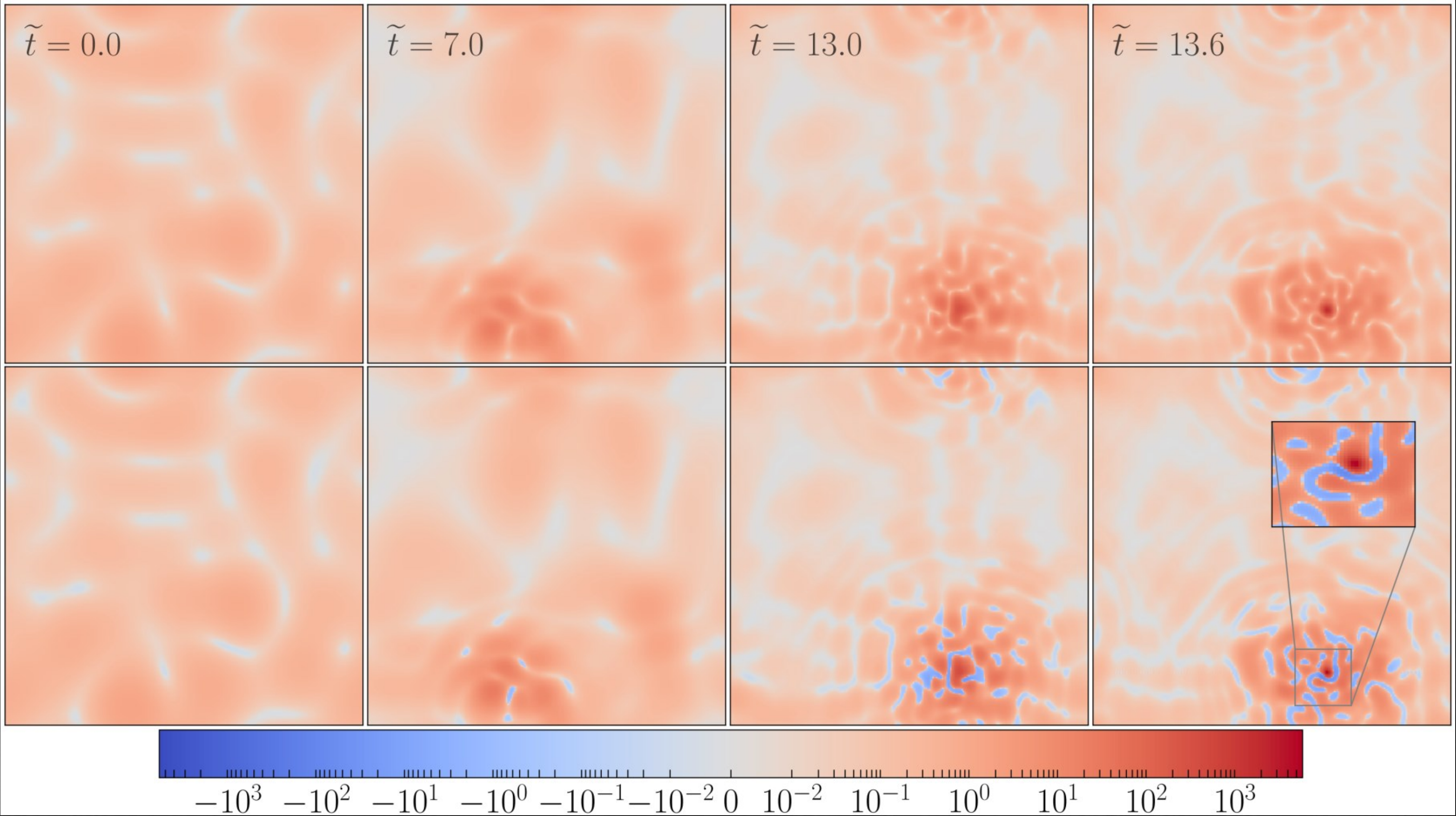
SP eqs can be generalized for vector DM and multicomponent DM

Nonminimally gravitating dark matter halos

With $\epsilon(mv_0L)^2 = -0.008$

$$\rho\frac{4\pi G}{m^2 v_0^4}$$

$$\rho_L\frac{4\pi G}{m^2 v_0^4}$$

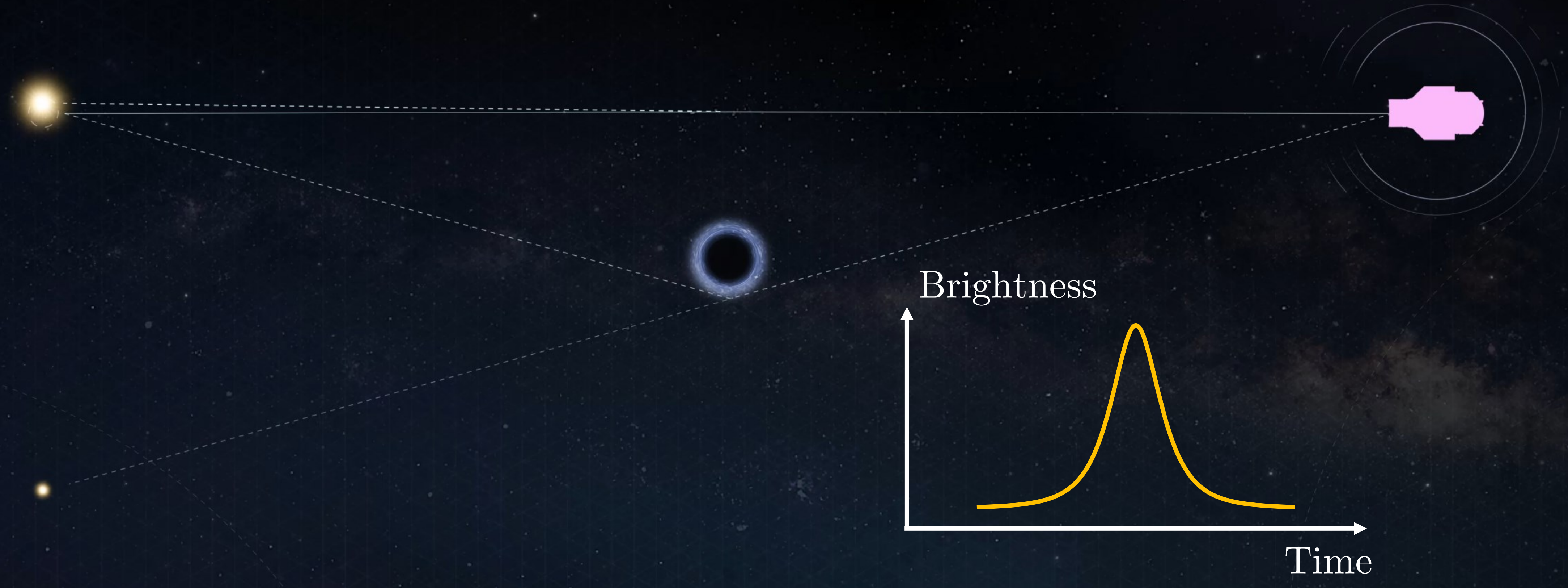


Regions with "negative density" emerge!

Chen & **HYZ** (JCAP, 2024)

Gravitational lensing

https://svs.gsfc.nasa.gov/20315/

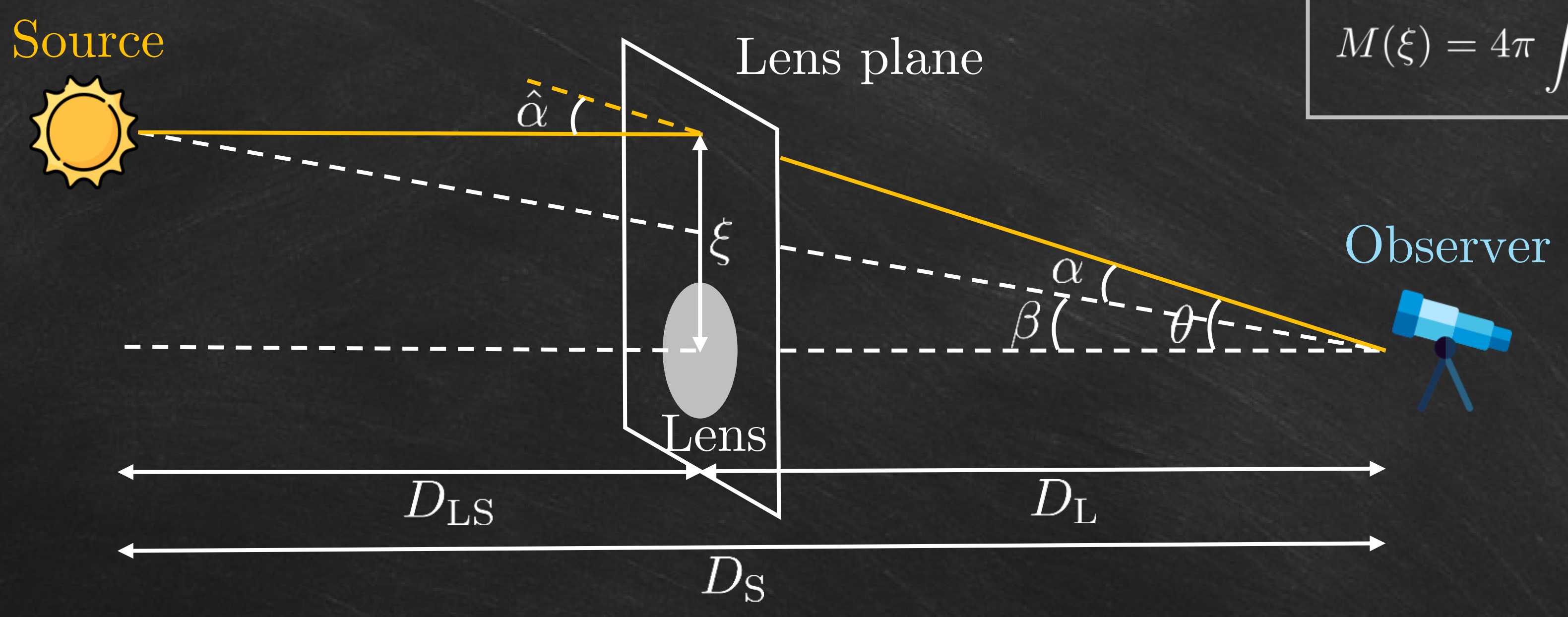


Microlensing in geometrical optics

$$\beta = \theta - \frac{D_{\rm LS}}{D_{\rm S}}\hat{\alpha}$$

$$\hat{\alpha} = \frac{4GM(\xi)}{\xi}$$

$$M(\xi) = 4\pi \int_0^{\xi} \xi' d\xi' \int_0^{\infty} dz' \rho_L(r')$$



Einstein ring $\quad \theta_{\rm E} = \sqrt{\frac{4GMD_{\rm LS}}{D_{\rm S}D_{\rm L}}} \sim 9 \times 10^{-4} \text{arcsec} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{10\text{kpc}}{D_{\rm S}}\right)^{1/2}$

Images are too close to be observationally resolved.
We can observe the change of brightness

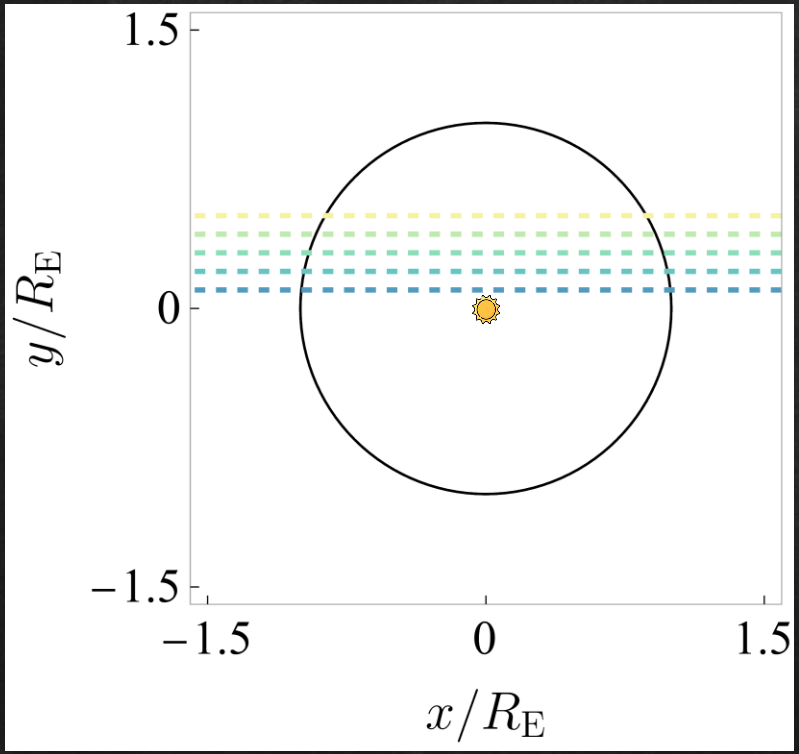
For reviews, see Meneghetti (Springer, 2021), Narayan & Bartelmann (astro-ph/9606001)

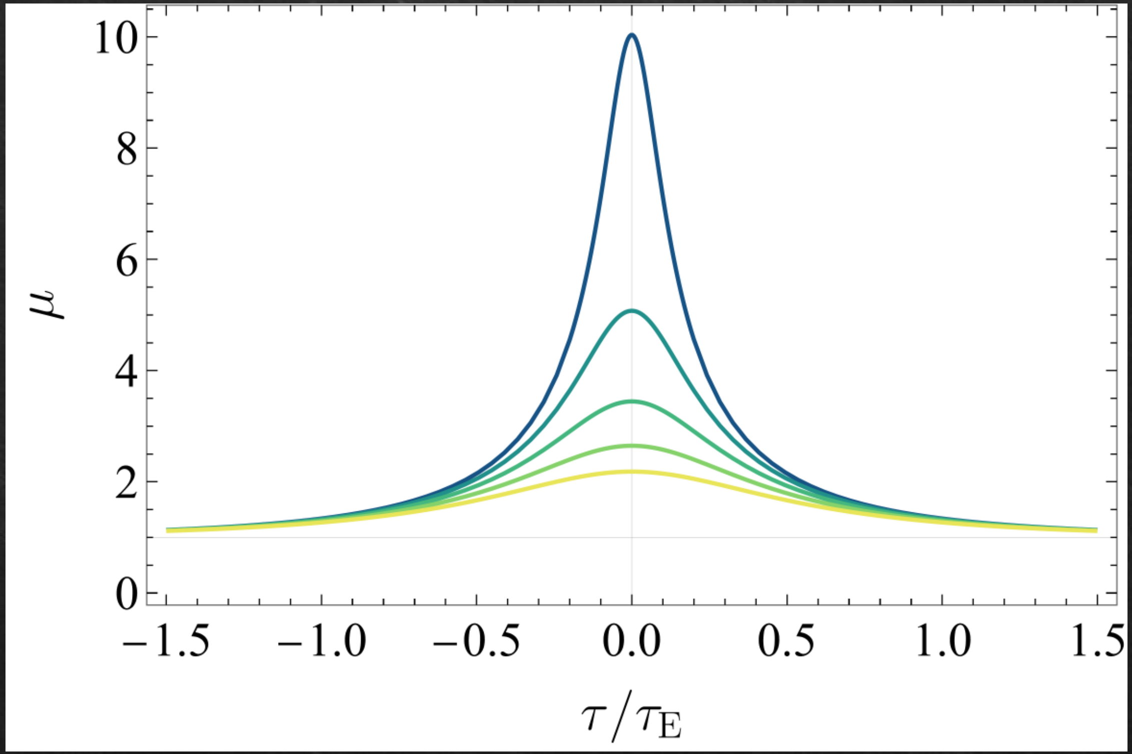
Light curves of a pointlike lens

Template for event selection

Different source trajectories on the lens plane

$$\mu = \frac{\text{Total flux with lensing}}{\text{Total flux w/o lensing}} = \frac{2+u^2}{u\sqrt{4+u^2}}$$



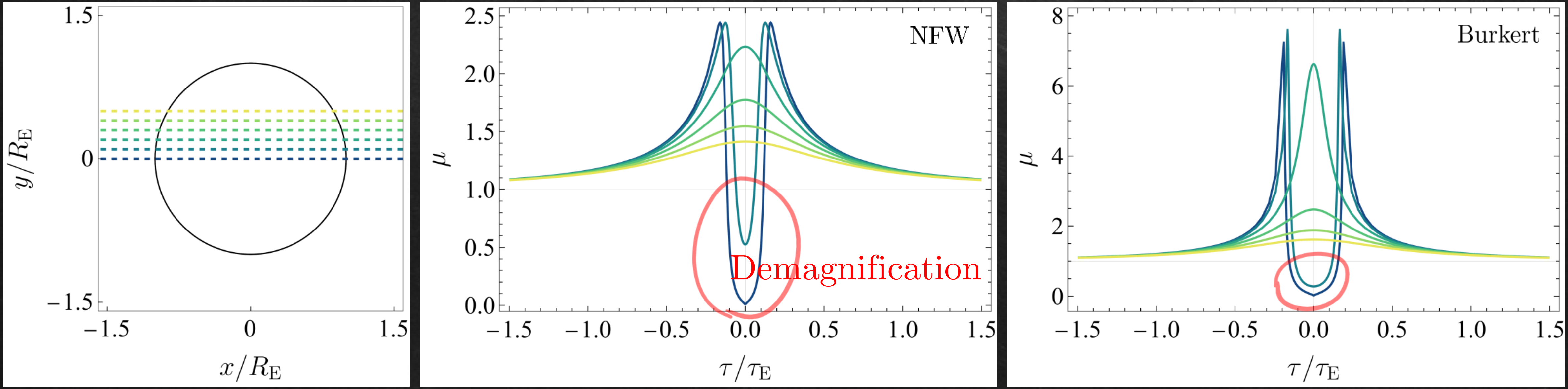


Gravity is attractive ➞ $\mu \geq 1$

Light curves with nonminimal couplings to gravity

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

$$\rho(r) = \frac{\rho_s}{(1+r/r_s)[1+(r/r_s)^2]}$$



Taking $\epsilon = -1, \ L = 0.6 r_s, \ R_E/r_s = 3$

HYZ (2510.05575)

Speed of gravitational waves

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -\frac{\xi_1}{2} R X_\mu X^\mu, \; -\frac{\xi_2}{2} R^{\mu\nu} X_\mu X_\nu$$

$$m^2 \epsilon L^2 = \xi_1 + \xi_2/2$$

Tensor mode action $\quad S^{(2)} = \frac{1}{2} \sum_{\lambda=+,\times} \int d^4x \, M_*^2 \left[\dot{h}_\lambda^2 - c_T^2 (\nabla h_\lambda)^2 \right]$

$$M_*^2 = M_{\rm P}^2 + \left(\xi_1 + \frac{2}{3}\xi_2 \right) \frac{\rho}{m^2}$$

$$M_*^2 c_T^2 = M_{\rm P}^2 + (\xi_1 + \xi_2) \frac{\rho}{m^2}$$

Density of the parallel component

Speed of GWs $\quad \alpha_T \equiv c_T^2 - 1 = \frac{\xi_2 \rho_{\hat{n}}}{m^2 M_{\rm P}^2} \simeq \frac{\xi_2 \rho}{3 m^2 M_{\rm P}^2}$

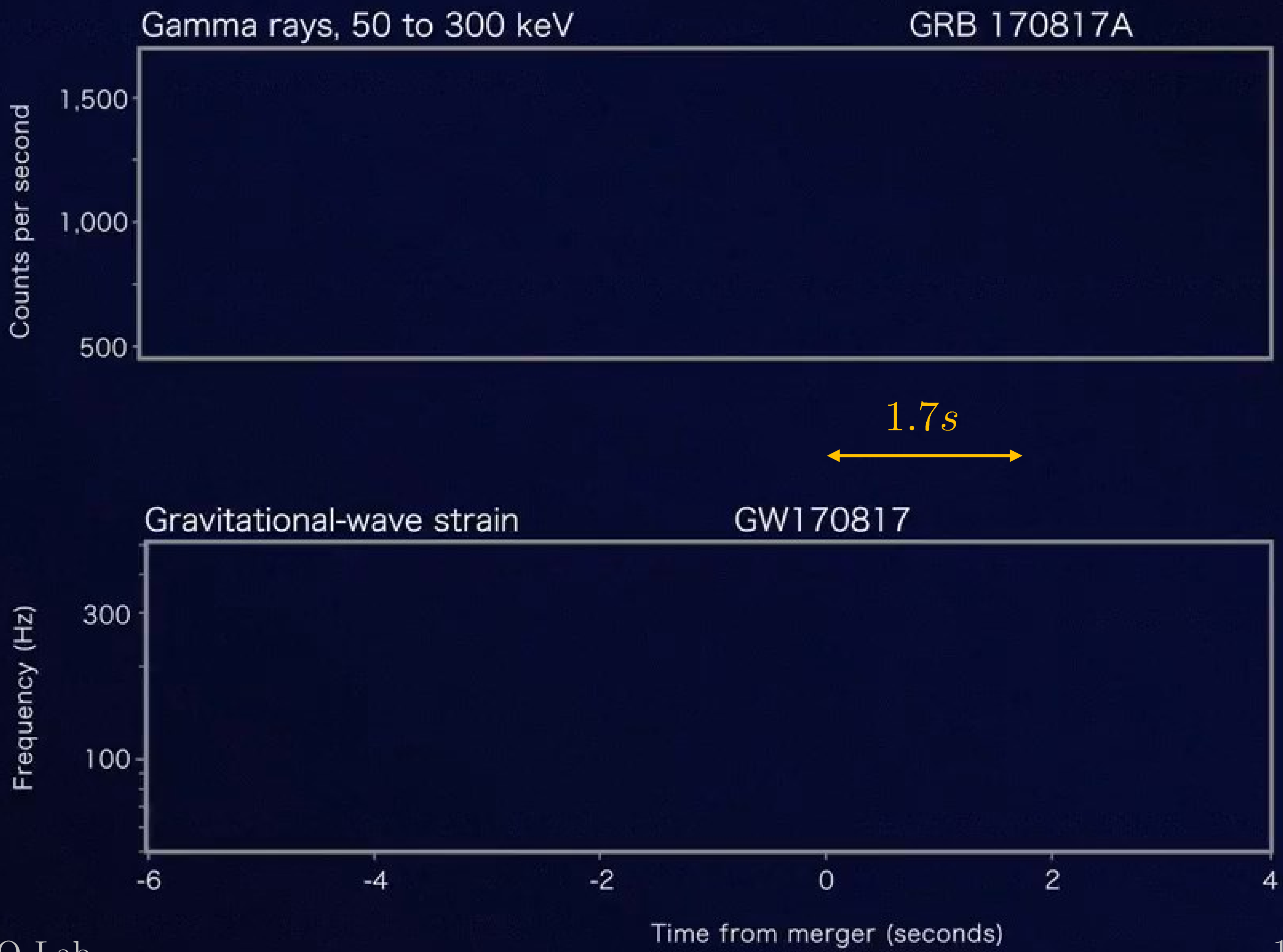
Positive ξ_2 leads to superluminal propagation

HYZ & Ling (JCAP, 2023)

For causality issues, see Liberati, et al. (AOP, 2002), Bruneton (PRD, 2007).

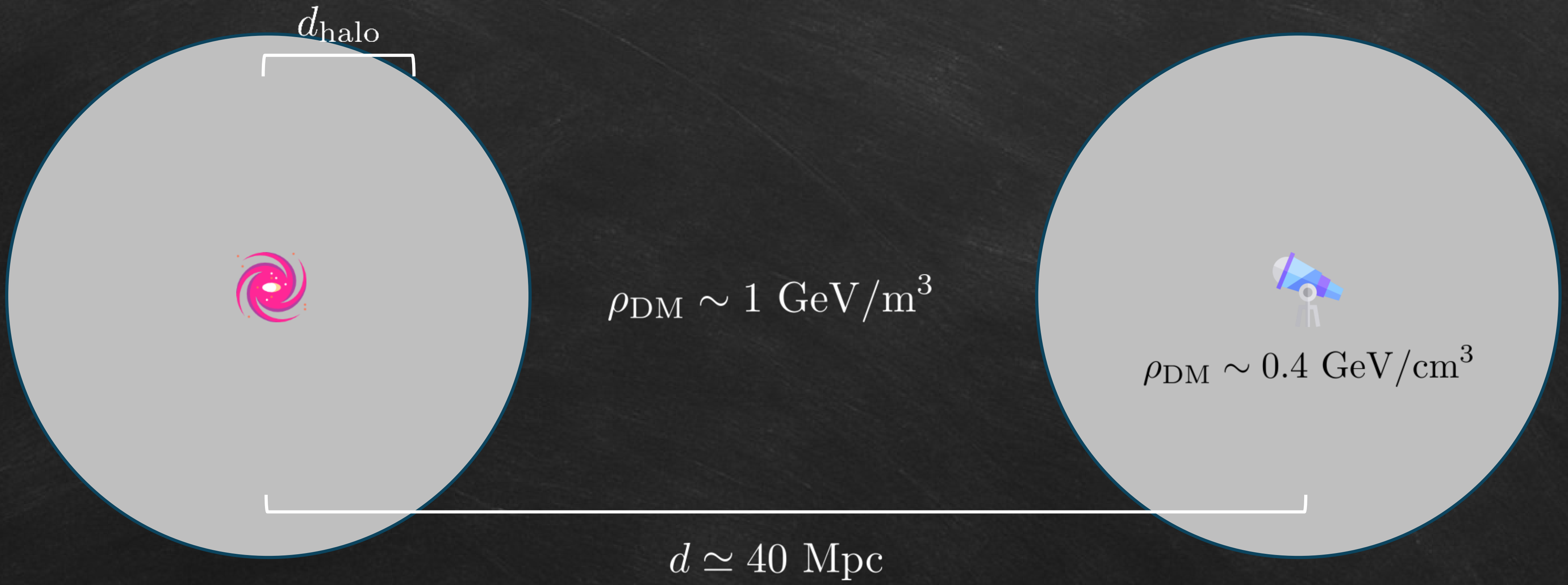
Fermi

LIGO



Credit: NASA GSFC & Caltech/MIT/LIGO Lab

Time difference is accumulated in halos



$$\Delta t_{\rm halo} \sim \frac{1}{2} d_{\rm halo} \alpha_T \simeq 8.89{\rm s} \left(\frac{d_{\rm halo}}{10{\rm kpc}}\right) \left(\frac{\alpha_T}{1.7 \times 10^{-11}}\right)$$

$$\Delta t_{\rm intga} \sim \frac{1}{2}(d - d_{\rm halo}) \alpha_T \simeq 0.098 \left(\frac{d - d_{\rm halo}}{40{\rm Mpc}}\right) \left(\frac{\alpha_T}{4.3 \times 10^{-17}}\right)$$

HYZ & Ling (JCAP, 2023)

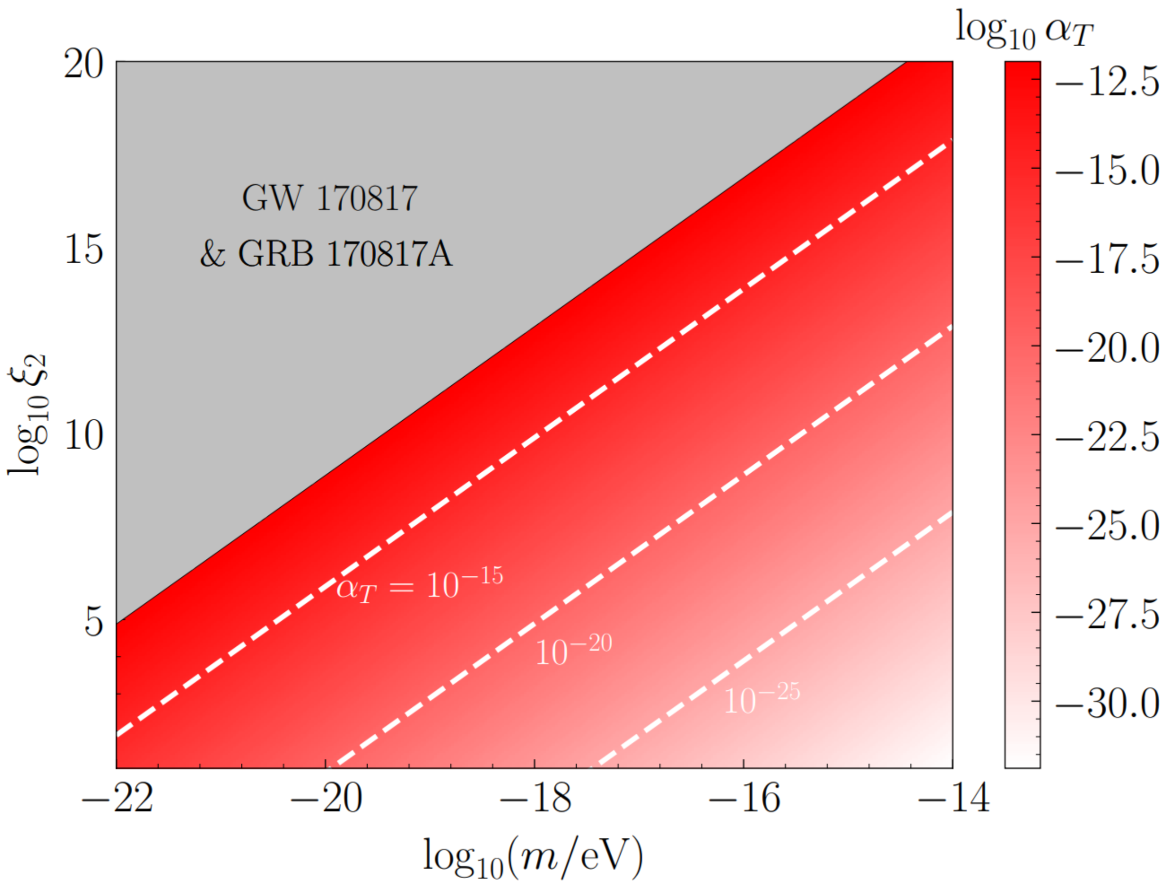
Constraints from GW 170817 & GRB 170817A

Constraints on GW speed (in halos):

$$-10^{-11} \lesssim \alpha_T \lesssim 10^{-12}$$

Relax the previous constraint obtained from the NS merger, $|\alpha_T| \lesssim 10^{-15}$

Baker, et al. (PRL, 2017)

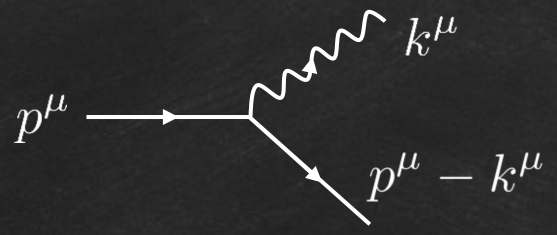


Assuming GRB peak was emitted after coalescence within 10s (conservative)

HYZ & Ling (JCAP, 2023)

Gravitational Cherenkov radiation

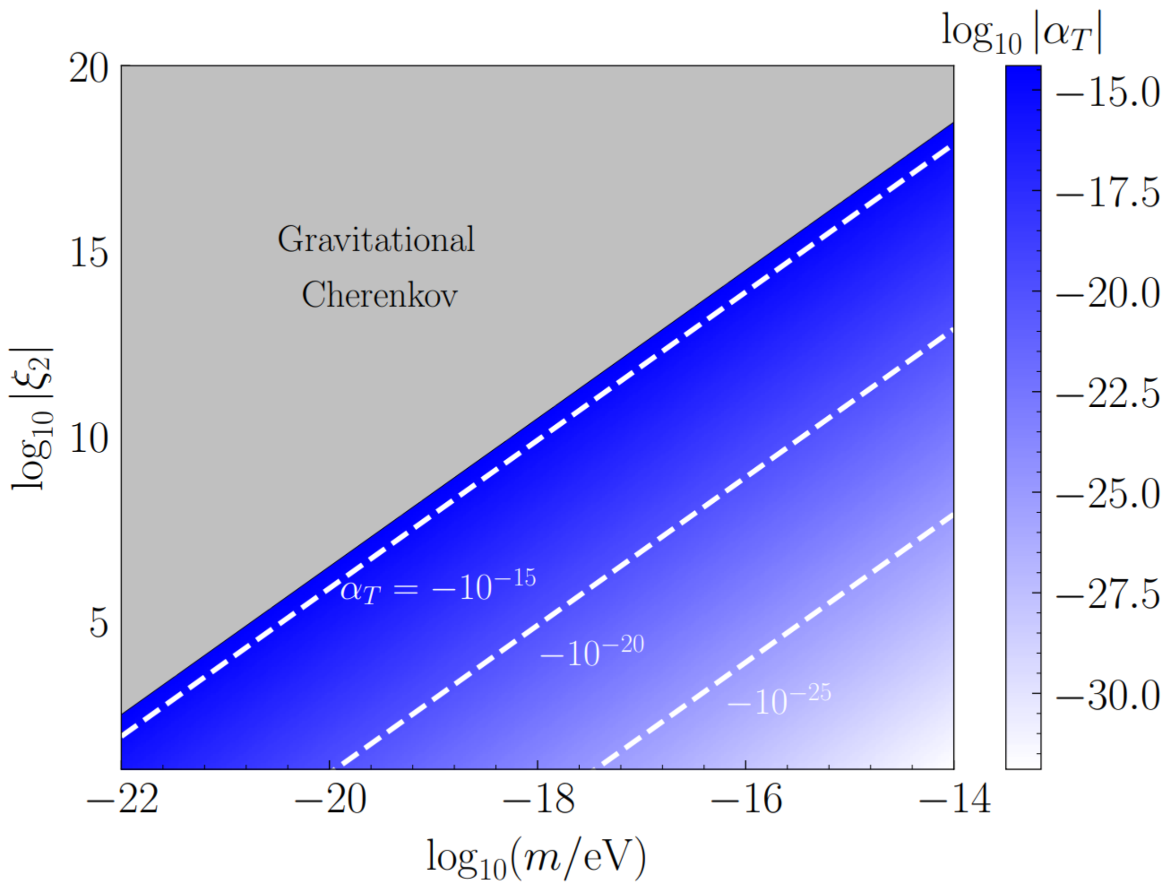
Cherenkov radiation



- Forbidden in vacuum
- In a medium, photon dispersion relation $|\boldsymbol{k}| = nk^0$
- For n>1, photons carry more momentum than energy, making Cherenkov radiation possible

Observation for 10^{11} GeV cosmic rays (OMG particle) with a galactic origin:

$$\alpha_T \gtrsim -4 \times 10^{-15}$$



Moore & Nelson (JHEP, 2001) ; **HYZ** & Ling (JCAP, 2023)

Summary

- EFT/quantum effects/pheno ➞ NMCs
- NMCs ➞ self-interactions + modified Newtonian gravity
- Astrophysical probes
  - DM structures with "negative" density — Zamani, et al. (EPJ C, 2025)
  - Strong and weak lensing (galaxies and clusters) ➞ $L \lesssim 100$ kpc
  - Microlensing (DM objects) ➞ Unique demagnification signals
  - Large-scale matter spectrum ➞ $L \lesssim 60$ pc — **HYZ** & Ling (JCAP, 2023)
  - GW speed (GW170817 + grav. Cherenkov) ➞ $L \lesssim 18$ pc